Optimal fast-response sliding-mode control for flexible-substrate measurement

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A R T I C L E   I N F O

Article history:
Received 22 February 2011
Accepted 14 January 2013
Available online 26 February 2013

Keywords:
Bending radius
Optimal fast-response sliding-mode control
Sliding-mode control
Linear quadratic estimator
Polyethylene terephthalate/indium tin oxide substrate

A B S T R A C T

In order to control bending radii of flexible substrates in displays, this paper presents an optimal fast-response sliding-mode control method. Maximum driving voltages are utilized in sliding-mode control based on a linear quadratic estimator. An optimal design for the threshold value and the reaching-law parameter are obtained by using a procedure of the fast-response regulation. Implementing the proposed method, this study measures electrical resistances for different widths of line patterns on polyethylene terephthalate/indium tin oxide substrates up to 11,000 bending times.

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1. Introduction

With rapid advance in semiconductor technology as well as in electronic display industry, the digital household appliances are becoming more and more diversified. However, hard electronic products can not satisfy people who demand comfortable and convenient lifestyle. The flexible electronics, which is thin and flexible, shock-resistant, and not limited by the occasion or space, will become the top choice of consumer electronic products. The inspection technique of flexible electronic products is not mature. Therefore, Grego, Lewis, Vick, and Temple (2005) provided two methods for bending-test technique. But there are some limiting factors when using the methods. The purpose of this study is to put forward a new control technique for measuring bending characteristics of flexible electronics under different bending radii by using a flexible-characteristics inspection system (FCIS). FCIS was designed to exert an external force on the flexible electronics. It resulted in a curved screen and made the flexible electronics crooked. Hence, this study shot the crooked states of flexible displays using a charge-coupled device (CCD), and carried out image processing using a LabVIEW software, calculated the bending radius of the flexible electronics (Gonzalez & Woods, 2008; Wen, Liu, Chen, Ko, & Chung, 2008). In mass and rapid inspections, there are various sizes and thin film layers in different flexible electronics products. Since parameters of FCIS mathematical model are different every time due to variable boundary conditions of each flexible electronics, the mathematical model is highly nonlinear. Therefore, in order to achieve stable bending-radius control, a controller is designed in the present study.

In constructing ARX models, according to Verhaegen and Verdult (2007), a cyclic manner of iteratively refining data from real-life measurements identifies model parameters for an unknown dynamical system. To carry out bending-radius control on FCIS in this study, a mathematical model of a FCIS mechanism is first identified by an ARX model (Peng, Ozaki, Toyoda, & Oda, 2001). Secondly, a control-algorithm design is developed according to results of FCIS plant identification. Simulation results obtained by CCD feedback signals are used to calculate the bending radius. Thirdly, the architecture of the control algorithm is integrated on FCIS to implement bending-radius control for measuring electrical characteristics of flexible electronics. Nevertheless, the mathematical model of FCIS is highly nonlinear, and there are also uncertainties and disturbance in FCIS. There are a lot of well-known control methods, e.g. fuzzy control, neural control, sliding-mode control (SMC) and so on. Compared to SMC, it takes a lot of time to obtain the parameters in neural control by multilayer neural network learning (Lin & Lee, 1999). Fuzzy control requires more system responses to design a good membership function. According to the experimental results (Chung, Wen, & Lin, 2007), the performance of SMC is better than fuzzy control in the presence of systemic uncertainties and disturbances. Because SMC is effective in dealing with systemic uncertainties and disturbances (Alfaro-Cid, McGookin, Murray-Smith, & Fossen, 2005; Edwards & Tan, 2006), an advanced SMC algorithm is developed in this study. In addition, iterative learning control (ILC) may improve the transient response and tracking...
performance of uncertain dynamic systems that operate repetitively (Xu, Panda, & Lee, 2009). According to a brief and categorization of ILC (Ahn, Chen, & Moore, 2007), there are two kinds of technical papers about ILC. One is related to the literature that focuses on ILC applications (Chen & Moore, 2002; Dou, Tan, Lee, & Zhou, 1999; Wu & Liu, 2004), and the other is related to the literature focused on the theoretical developments (Chien, 2000; Chen, Moore, & Bahl, 2004; Jiang & Unbehauen, 2002). Briefly, ILC is an intelligent control methodology (Abdallah, Soualian, & Schamiloğlu, 1998) for improving the transient performance of systems that operate repetitively over a fixed time interval. Messner, Horowitz, Kao, and Boals (1991) compensated the nonlinear properties of plant and disturbance by using learning estimation. However, iterative learning estimations require more time to compute control inputs. As a result, to achieve a robust and fast-response control system for various kinds of flexible electronics, an optimal fast-response sliding-mode control (OFRSMC) is presented in this study with simulations and experiments for measuring electrical characteristics of flexible electronics. The new technique measures electrical characteristics of flexible polyethylene terephthalate (PET)/indium tin oxide (ITO) substrates up to 11,000 bending times from flat to 2 cm bending radius by using FCIS based on the OFRSMC. In addition, this study measures resistances for 5, 10, 20, 30, 40, and 50 dpi widths of the line pattern on PET/ITO samples. If resistances of flexible electronics increase a lot after bending tests, flexible electronics probably will not work. According to inspection results, a designer or maker of flexible electronics can design useful and comfortable flexible electronic products for human being.

2. Measurement apparatus of flexible-characteristics inspection system

In order to measure electrical characteristics of the flexible electronics, this study puts forward the flexible-characteristics inspection system, as depicted in Fig. 1, which comprises a clip unit, a flexible-characteristics inspection stage with a motor, a motion controller, a computer, and a CCD camera. In addition, the clip unit is configured with two clipping arms which can work cooperatively for holding a flexible electronic substrate. In order to bend the flexible substrate in pure bending, this study designed an opposite moving of the two clipping arms in the round moving simultaneously to implement. The sever motor drives the flat belt pulley and the spur gear pair in the same speed simultaneously. The flat belt pulley and the spur gear pair link up different clipping arms, respectively. The transmission speed from the server motor to flat belt pulley is the same as to spur gear pair, but in opposite directions respectively. Therefore, both clipping arms can make an opposite movement of the two clipping arms in the circular path at the same moving speed simultaneously. Moreover, because there is only a bending force on the perpendicular direction of the flexible substrate by clipping arms, an opposite movement of the two clipping arms in the circular path simultaneously can make a pure bending for testing the flexible substrate.

Moreover, to measure the electrical-resistance characteristics, a 4-point-probe electrical-resistance measurement apparatus is integrated into two clipping arms, as shown in Fig. 2. Measurement errors concerning the electrode-contact electrical-resistance can hence be reduced by using the principle of the 4-point-probe electrical-resistance measurement (Meier & Levinzon, 1965). In this study, the flexible substrate is sheet-like or plate-like object.

3. Design and simulation of optimal fast-response sliding-mode control

3.1. Design of optimal fast-response sliding-mode control

FCIS depicted in Fig. 1 is a nonlinear system. Real-world nonlinear control problems are dealt with by different techniques (Cheng & Li, 1998; Li & Shieh, 2000). The model of FCIS can be
are prescribed set for the sliding vector design in the proposed sliding-mode control:

1. Re{\lambda_i} < 0, \forall j \in \mathbb{R}, \forall j < 0, \forall j \neq \lambda_i.
2. Any eigenvalue in \{\lambda_1, \ldots, \lambda_m\} is not in the spectrum of \text{A}_z.
3. The number of any repeated eigenvalues in \{\lambda_1, \ldots, \lambda_m\} is not greater than \text{m}, the rank of \text{B}_z, where \{\lambda_1, \lambda_2, \ldots, \lambda_n, \ldots, \lambda_m\} are sliding-mode eigenvalues and \{\lambda_1, \lambda_2, \ldots, \lambda_m\} are virtual eigenvalues.

As proved by Sinswat and Fallside (1977), if the condition (3) in the above is established, the control system matrix \text{A}_z-\text{B}_z\text{K} can be diagonalized as

\[ \text{A}_z-\text{B}_z\text{K} = \begin{bmatrix} V^{-1} & 0 \\ 0 & \Gamma \end{bmatrix} \begin{bmatrix} \\ V \end{bmatrix} \]

(3)

where \text{\Phi}_\text{V} = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_m], \Gamma = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_m], and \text{V} and \text{F} are left eigenvectors with respect to \text{\Phi}_\text{V} and \Gamma, respectively. Hence, Eq. (3) can be rewritten as

\[
\begin{align*}
\text{V}(\text{A}_z-\text{B}_z\text{K}) &= \text{\Phi}_\text{V}\text{V} \\
\text{F}(\text{A}_z-\text{B}_z\text{K}) &= \Gamma \text{F}
\end{align*}
\]

(4)

rearrangement of Eq. (4) yields

\[
\text{F}(\text{A}_z-\text{I}\text{F}) = \text{rank}(\text{FB}_z)\text{K}
\]

(5)

according to Chang (1999),

\[
\text{rank}(\text{FA}_z-\text{I}\text{F}) = \text{rank}(\text{F})
\]

(6)

since \text{F} contains \text{m} independent left eigenvectors, one has \text{rank}(\text{F}) = \text{m}. From Eqs. (5) and (6), it is also true that \text{rank}(\text{FA}_z-\text{I}\text{F}) = \text{rank}(\text{FB}_z)\text{K} = \text{rank}(\text{F}) = \text{m}. In other words, \text{FB}_z is invertible. With the designed left eigenvector \text{F} above, the sliding function \text{S}(k) is designed as

\[
\text{S}(k) = \text{FX}(k)
\]

(7)

The second step is the discrete-time switching control design. A different and much more expedient approach than that of Gao and Hung (1993) is adopted here. This approach is called the reaching law approach that has been proposed for continuous variable structure control (VSC) systems (Hung, Gao, & Hung, 1993). This control law is synthesized from the reaching law in conjunction with a plant model and the known bounds of perturbations. For a discrete-time system, the reaching law is (Gao, Wang, & Homaifa, 1995)

\[
\text{S}(k+1) - \text{S}(k) = -q\text{T}\text{S}(k) - \varepsilon T\text{sgn}(\text{S}(k))
\]

(8)

where \text{T} > 0 is the sampling period, \text{q} > 0 is the reaching-law parameter, \varepsilon > 0 is the sliding-layer thickness and 1–qT > 0. Therefore, the switching control law for the discrete-time system is derived based on this reaching law. From Eq. (7), \text{S}(k) and \text{S}(k+1) can be obtained in terms of sliding vector \text{F} as

\[
\begin{align*}
\text{S}(k) &= \text{FX}(k) \\
\text{S}(k+1) &= \text{FX}(k+1) = \text{FA}_z\text{X}(k) + \text{FB}_z\text{U}(k)
\end{align*}
\]

(9)

it follows that:

\[
\text{S}(k+1) - \text{S}(k) = \text{FA}_z\text{X}(k) + \text{FB}_z\text{U}(k) - \text{FX}(k)
\]

from Eqs. (8) and (10),

\[
\text{S}(k+1) - \text{S}(k) = -qT\text{S}(k) - \varepsilon T\text{sgn}(\text{S}(k)) = \text{FA}_z\text{X}(k) + \text{FB}_z\text{U}(k) - \text{FX}(k)
\]

solving for \text{u}_e(k) gives the switching control law

\[
\text{u}_e(k) = -(\text{FB}_z)^{-1}[\text{FA}_z\text{X}(k) + (qT-1)\text{FX}(k) + \varepsilon T\text{sgn}(\text{FX}(k))]
\]

(11)
In order to achieve the output tracking control, a reference command input $r(k)$ is introduced into the system by modifying a state feedback control law $u_R(k) = -KX(k)$ with pole-placement design method \cite{Franklin, Powell, Workman 1998} to become

$$u_R(k) = N_u r(k) - K(X(k) - N_x r(k))$$ \tag{12}

where $K \in \mathbb{R}^{n}$ is a gain matrix obtained by assigning $n$ desired eigenvalues $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ of $A_2 - B_2 K$ and

$$
\begin{bmatrix}
  A_2 - I & B_2 \\
  C_2 & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
  0 \\
  1
\end{bmatrix}
$$ \tag{13}

The proposed SMC input, based on Eq. (12), is assumed to be

$$u_S(k) = u_R(k) + u_{sc}(k) = N_u r(k) - K(X(k) - N_x r(k)) + u_{sc}(k)$$ \tag{14}

substituting Eq. (11) into (14) gives the proposed SMC input as

$$u_S(k) = N_u r(k) - K(X(k) - N_x r(k)) - (FB_z)^{-1}[FA_z X(k) + (q_T - 1) FX(k) + e T sgn(FX(k))]$$ \tag{15}

The pole-placement SMC design method utilizes the feedback of all the state variables to form the desired sliding vector. In practice, not all the state variables are available for direct measurement. It is necessary to estimate the state variables that are not directly measurable. Therefore, the SMC input depicted in Eq. (15) is rewritten as

$$u_S(k) = N_u r(k) - K(X(k) - N_x r(k)) - (FB_z)^{-1}[FA_z X(k) + (q_T - 1) FX(k) + e T sgn(FX(k))]$$ \tag{16}

where $\tilde{X}(k)$ is an observed state. Define a full-order estimator (FOE) as

$$\hat{X}(k+1) = A_2 \hat{X}(k) + B_2 u(k) + K_c(y(k) - C_2 \hat{X}(k))$$ \tag{17}

and $K_c$ is an observer feedback gain matrix.

Image noise in CCD feedback signals is an error factor in controlling a bending radius. Hence, a linear quadratic estimator (LQE) is applied here to estimate optimal states. Based on Eq. (2), consider a system model

$$\begin{bmatrix}
  X(k+1) \\
  y(k)
\end{bmatrix} =
\begin{bmatrix}
  A_2 & B_2 \\
  C_2 & 0
\end{bmatrix} \begin{bmatrix}
  u(k) \\
  \alpha(k)
\end{bmatrix}$$ \tag{18}

where $X(k) \in \mathbb{R}^n$ is the state variable, $u(k) \in \mathbb{R}^m$ is the control input voltage, $y(k) \in \mathbb{R}^m$ is the assumed plant output related to the bending radius, and $\alpha(k) \in \mathbb{R}^m$ and $\theta(k) \in \mathbb{R}^m$ are system disturbances and measurement noise with covariances $E[\alpha(k)\alpha^T(k)] = Q$, $E[y(k)y^T(k)] = R$, and $E[\alpha(k)y^T(k)] = 0$. The objective of LQE is to find a vector $\hat{X}(k)$, which is an optimal estimation of the present state $X(k)$. Here “optimal” means the cost function \cite{Franklin, Phillips 1995}

$$J = \lim_{T \rightarrow \infty} E \left\{ \int_0^T (X^T Q X + u^T R u) dt \right\}$$ \tag{19}

is minimized. The solution is an estimator written as

$$\begin{bmatrix}
  \dot{\hat{X}}(k+1) \\
  \dot{y}(k)
\end{bmatrix} =
\begin{bmatrix}
  A_2 & B_2 \\
  C_2 & 0
\end{bmatrix} \begin{bmatrix}
  u(k) \\
  \alpha(k)
\end{bmatrix}$$ \tag{20}

where $K_f$ is the “optimal Kalman” gain $K_f = PC_f R^{-1}$ and $P$ is the solution of the algebraic Riccati equation

$$A_2 P + P A_2^T - P C_f R^{-1} C_f P + Q = 0$$ \tag{21}

SMC and LQE are integrated into a so called optimal sliding-mode control (OSMC) with control input

$$u_{os}(k) = N_u r(k) - K(X(k) - N_x r(k)) - (FB_z)^{-1}[FA_z \hat{X}(k) + (q_T - 1) FX(k) + e T sgn(FX(k))]$$ \tag{22}

In order to expedite the response time, the maximum driving voltage control is utilized in OSMC by a fast-response regulator, whose input is written as

$$u_R(k) = \frac{1}{2} U_{\text{max}} \left[ \text{sgn}(r(k) - \theta) + 1 \right] - \frac{1}{2} \left[ \text{sgn}(r(k) - \Phi) - 1 \right]$$

$$[N_u r(k) - K(X(k) - N_x r(k)) - (FB_z)^{-1}[FA_z \hat{X}(k) + (q_T - 1) FX(k) + e T sgn(FX(k))]$$ \tag{23}

where $\Phi$ is a threshold value of the reference command input. When $|r(k)|$ is larger than $\Phi$, $\text{sgn}(r(k) - \Phi) = -1$. Therefore, the proposed input is

$$u_R(k) = \frac{1}{2} U_{\text{max}} [1 + 1 - \frac{1}{2} (1 - 1)] [N_u r(k) - K(X(k) - N_x r(k)) - (FB_z)^{-1}[FA_z X(k) + (q_T - 1) FX(k) + e T sgn(FX(k))]]$$ \tag{24}

as a consequence, the maximum driving voltage $U_{\text{max}}$ is applied. However, when $|r(k)|$ is less than $\Phi$, $\text{sgn}(r(k) - \Phi) = -1$. Therefore, the proposed input becomes

$$u_R(k) = \frac{1}{2} U_{\text{max}} [1 + 1 - \frac{1}{2} (1 - 1)] [N_u r(k) - K(X(k) - N_x r(k)) - (FB_z)^{-1}[FA_z X(k) + (q_T - 1) FX(k) + e T sgn(FX(k))]]$$ \tag{25}

In Eq. (23), both threshold value $\Phi$ and reaching-law parameter $q$ affect response speed. In addition, boundary conditions of each flexible electronics are different and the mathematical model is a nonlinear model for controlling the bending radius. In order to achieve a fast-response and robust bending-radius control, an optimal design for $\Phi$ and $q$ is vital. Therefore, fast response regulation is developed in this study to obtain $\Phi$ and $q$ optimally, so that the maximum driving voltage is applied in the early control period. Assume a nonlinear output written as

$$y(k) = \alpha e^{ak}$$ \tag{26}

taking the logarithm to both sides of Eq. (26) yields

$$\ln y(k) = \ln a + bk$$ \tag{27}

the best-fit values \cite{Perl, 1960} are thus

$$a = e^{n \sum_{k=1}^n \ln y(k)}$$ \tag{28}

$$b = \frac{n \sum_{k=1}^n k \sum_{k=1}^n \ln y(k)}{n \sum_{k=1}^n k^2}$$ \tag{29}

in terms of $a$ and $b$ parameters, the predictive arrival time $k_r$ is obtained as

$$k_r = \frac{\ln R - \ln a}{b}$$ \tag{30}

where $R_i$ is a command input. Then, the threshold value for the maximum driving voltage input is obtained as

$$\Phi = \alpha e^{\theta k}$$ \tag{31}

where $s_i$ is a ratio parameter for minimum sliding-mode time and the threshold value has to be no smaller than $\varepsilon_T$. Then, reaching-law parameter $q$ is also obtained as

$$q = \frac{1 - \Phi}{T}$$ \tag{32}

therefore, according to a procedure of the fast-response regulation from Eq. (28) to (32), the threshold value and reaching-law parameter can be obtained. The system block diagram is shown in Fig. 3 where $R_i$ is an input signal of the bending radius. In the beginning, the maximum driving voltage is applied to control a bending radius. A CCD provides the bending-radius feedback signal for LQE, which can estimate optimal states in the presence of system disturbances and measurement noise in SMC. After
images, as shown in Fig. 1. The sampling time and bending radii of substrates are measured in real time by CCD
0.2 Hz for bending frequencies from negative to positive bending, ARX model. A motor provides driving frequencies varying up to
nonlinear control problem, the model of FCIS is identified by an
Cz
bending radius
the transfer function that relates the control voltage
Verdult, 2007) from bending-radius measurements. Accordingly,
by using a cyclic manner of iteratively refining data (Verhaegen &
3.2. Stability analysis of OFRSMC
This section deals with a system model described by Eq. (2)
and defines a reference command input r(k). To overcome the
nonlinear control problem, the model of FCIS is identified by an
ARX model. A motor provides driving frequencies varying up to
0.2 Hz for bending frequencies from negative to positive bending,
and bending radii of substrates are measured in real time by CCD
images, as shown in Fig. 1. The sampling time T is 0.08 s. All the
parameters of the FCIS described by the ARX model are obtained
by using a cyclic manner of iteratively refining data (Verhaegen &
Venud, 2007) from bending-radius measurements. Accordingly,
the transfer function that relates the control voltage \( V(z) \) and the bending radius \( Y(z) \) can be written as

\[
H(z) = \frac{Y(z)}{V(z)} = \frac{0.10361z^{-2} - 0.14046z + 0.06742}{z^2} \tag{34}
\]

in addition, the transfer function of FCIS can be rewritten in state-
variables form. Accordingly, \( A_z = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \), \( B_z = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \) and
\( C_z = \begin{bmatrix} 0.10361 & -0.14046 & 0.06742 \end{bmatrix} \) are obtained for Eq. (2).

In this study, Ackermann’s formula (Ackermann, 1972) is
used to determine the pole-placement feedback gain matrix
\( K = \begin{bmatrix} -0.08 & 0.0017 & 0.00001 \end{bmatrix} \). The pole-placement algorithm
described in Section 3.1 is utilized to determine a sliding vector
\( F = \begin{bmatrix} 0.99756 & -0.06983 & 0.001 \end{bmatrix} \). In practice, the fact that not all
state variables are available for direct measurement results in the
necessity to estimate the state variables that are not measurable.
Hence, the FOE designed by Ackermann’s formula and LQE are
utilized in this study, where the observer feedback gain matrix is
\( K_e = \begin{bmatrix} -0.05933 \\ 6.0022 \\ -6.8346 \end{bmatrix} \) and the optimal Kalman gain is
\( K_f = \begin{bmatrix} 1.9193 \\ -1.7375 \\ 0.66094 \end{bmatrix} \), respectively. In addition, according to Eq. (13),
\( N_u \) and \( N_x \) are obtained as \( \begin{bmatrix} 32.712 \\ 32.712 \\ 32.712 \end{bmatrix} \), respectively.

In order to analyze the stability of FCIS with the OFRSMC,
Lyapunov analysis (Slotine & Li, 1991) and a proof method by
contradiction (Andrilli & Hecker, 2009) is utilized. Consider a
discrete-time quadratic Lyapunov function candidate
\( V_i(x(k)) = X^*(k)\Omega X(k) \) \tag{35}
where \( \Omega \) is a given symmetric positive definite matrix. Differentiating the positive definite function \( V_i(x(k)) \) along the system
trajectory yields another quadratic form
\( \Delta V_i(x(k)) = V_i(x(k+1)) - V_i(x(k)) = X^*(k)A_x^*\Omega A_x-\Omega X(k) \)
\( = -X^*(k)A_x X(k) \) \tag{36}
where
\( A_x^*\Omega A_x-\Omega = -A \) \tag{37}
is called a Lyapunov equation and $A_i$ is a system state matrix with a control method. If both $\Omega$ and $\Lambda$ are both positive definite matrices satisfying Eq. (37), the system is globally asymptotically stable. Therefore, according to the OFRSMC law, when $|\hat{r}(k)|$ is larger than $\Phi$, Eq. (33) is rewritten as
\begin{equation}
\phi_L(k) = \phi_{\text{max}}
\end{equation}
(38)
substituting Eq. (38) into (2) gives the model of FCIS with the OFRSMC as
\begin{align}
X(k+1) &= A_1X(k) + B_2U_{\text{max}} \\
\hat{y}(k) &= C_2\hat{X}(k)
\end{align}
(39)
where $A_1 = A_i$. According to a proof method by contradiction, firstly assume that Eq. (39) is unstable. Eq. (37) is rewritten as
\begin{equation}
A_i^*\Lambda A_i - \Lambda = -\Phi
\end{equation}
(40)
let $\Lambda = I$ and denote $\Omega$ as a symmetric matrix. The solution $\Omega$ in Eq. (40) is
\begin{equation}
\begin{bmatrix}
1 & 6.6613e^{-16} & 1.6177e^{-16} \\
6.6613e^{-16} & 2 & 2.433e^{-15} \\
1.6177e^{-16} & 2.433e^{-15} & 3
\end{bmatrix}
\end{equation}
According to Sylvester’s criterion (Ogata, 1994), $\Omega$ is positive definite. The solution $\Omega$ contradicts the assumption that the model of FCIS with the OFRSMC when $|\hat{r}(k)| - \Phi > 0$ is unstable.

On the other hand, when $|\hat{r}(k)|$ is less than $\Phi$, Eq. (33) is rewritten as
\begin{equation}
\phi_{\text{OSM}}(k) = -K\hat{X}(k) - (FB_2)^{-1}FA_1\hat{X}(k) = -(K + (FB_2)^{-1}FA_2)\hat{X}(k)
\end{equation}
(41)
where $F, G \geq 1$ and $eT \to 0$. Substituting Eq. (41) into (2) gives the model of FCIS with the OFRSMC as
\begin{align}
\hat{X}(k+1) &= A_1\hat{X}(k) - B_2(K + (FB_2)^{-1}FA_2)\hat{X}(k) = (A_1 - B_2(K + (FB_2)^{-1}FA_2))\hat{X}(k) \\
\hat{y}(k) &= C_2\hat{X}(k)
\end{align}
(42)
according to a proof method by contradiction, assume that Eq. (42) is unstable, Eq. (37) is rewritten as
\begin{equation}
(A_1 - B_2(K + (FB_2)^{-1}FA_2))\Lambda A_i - \Lambda = -\Phi
\end{equation}
(43)
let $\Lambda = I$ and denote $\Omega$ as a symmetric matrix. The solution $\Omega$ in
where $e(t)$ is an error function of the plant and $T_F$ is a finite time (Franklin et al., 1998). The integral of time multiplied by absolute error (ITAE) that provides the performance index of the best sensitivity is expressed as

$$\text{ITAE} = \int_0^{T_F} t |e(t)| dt$$

where $e(t)$ is an error function of the plant and $T_F$ is a finite time (Franklin et al., 1998). According to the simulated results shown in Figs. 4 and 5, the performance of three control systems in controlling the flexible substrate at 0.5 cm$^{-1}$ bending curvature are evaluated by using IAE and ITAE indices and the results, are shown in Table 1.

In addition, Fig. 6 depicts simulation results of FCIS with random measurement noise in controlling the flexible substrate at 0.25 cm$^{-1}$ bending curvature. The control performances of three control systems are evaluated by using IAE and ITAE indices and the results, are shown in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Without random measurement noise</th>
<th>With random measurement noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAE</td>
<td>ITAE</td>
</tr>
<tr>
<td>SMC with FOE</td>
<td>7.3082</td>
</tr>
<tr>
<td>OSMC</td>
<td>7.3082</td>
</tr>
<tr>
<td>OFRSMC</td>
<td>5.2747</td>
</tr>
</tbody>
</table>

The following conclusions can be arrived at from the analysis of simulation results from Figs. 4 to 6 and Table 1:

1. According to Fig. 4, there are no overshoots without random measurement noise by using the three controllers. Because in controlling bending radius the overshoot situation is not allowed. We succeed in designing the three controllers. However, according to Figs. 5 and 6, there are overshoots in the control performances by using SMC with FOE controller. In Table 1, the control performances without and with random measurement noise based on SMC with FOE controller are obviously worse than the other two. Accordingly, the control performance of SMC with LQE is better than SMC with FOE in overcoming measurement noise.

2. In Fig. 6, one bending cycle using either SMC with FOE or OSMC needs 12 s, whereas using the OFRSMC needs 8 s only. Accordingly, there are more bending cycles in 36 s by using the OFRSMC method than by using SMC with FOE and OSMC.

3. From simulation results, the response time for the bending-radius control based on the OFRSMC is the shortest among the three controllers. And the response time by using the OFRSMC is almost half time by using SMC with FOE or OSMC. In Table 1, the control performances based on the OFRSMC are obviously better than the other two. Therefore, the bending-radius on FCIS based on the OFRSMC performs better than that based on SMC with FOE, or the OSMC. It is certain that the OFRSMC method is capable of fast controlling the bending radius of flexible substrate on FCIS robustly and successfully. Section 4 depicts experimental results of controlling the bending radius of flexible substrates on FCIS based on SMC with FOE, OSMC, and OFRSMC, respectively.

Fig. 6. Simulation results of FCIS with random measurement noise in controlling the flexible substrate at 0.25 cm$^{-1}$ bending curvature. Blue solid lines represent command input, and red solid lines are the system output based on (a) SMC with FOE, (b) OSMC and (c) OFRSMC. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
4. Experimental results

4.1. Experimental results of OFRSMC

In order to measure electrical characteristics of flexible electronics under different radii of curvature, a commercially available ITO-coated PET (OCTM 100) from CPFilms Inc. is utilized. A laser writer is utilized to make line patterns of different widths on a PET/ITO sample for a flexible back plate, which is used to drive the display media. SMC with FOE, OSMC, and OFRSMC control the bending radius at 2 cm respectively and the bending-radius control results are shown in Fig. 7. In addition, Fig. 8 compares experimental results of FCIS in controlling the flexible substrate at 4 cm bending radius based on SMC with the FOE, OSMC, and OFRSMC repeatedly. In experiments, parameters of SMC with FOE and OSMC in simulations are utilized. In the experiment based on OFRSMC, the maximum driving voltage is applied in the first five control cycles. By using five control-cycle data and the fast-response regulation, $a=0.072$, $b=0.566$, $k_r=3.424$, $s_r=0.3$, the threshold value $\phi=0.129$ and the reaching-law parameter $q=10.89$ are obtained. According to the experimental results shown in Fig. 7, the performances of the three control systems in controlling the flexible substrate at 2 cm bending radius are evaluated by using IAE and ITAE indices and the results are shown in Table 2.

The following conclusions are drawn from experimental results:

1. According to Fig. 7 and Table 2, there appears no overshoot by using the three controllers. The performance of bending-radius control based on the OFRSMC is better than that based on SMC with FOE and OSMC, because the OFRSMC is more effective in overcoming system disturbances and measurement noise.

2. In Fig. 8, three-cycle bending operations by using SMC with FOE, OSMC, and the OFRSMC need 37 s, 36 s, and 17 s, respectively. Due to fast-response-regulation capability of the OFRSMC, the three-cycle bending period based on the OFRSMC is the shortest among the three controllers.

3. In a manner similar to simulation results, the OFRSMC response is the fastest and the most robust in view of Figs. 7 and 8 and Table 2. Compared to SMC with FOE and

Table 2

<table>
<thead>
<tr>
<th>Control System</th>
<th>IAE</th>
<th>ITAE</th>
</tr>
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<tbody>
<tr>
<td>SMC with FOE</td>
<td>183.95</td>
<td>264.66</td>
</tr>
<tr>
<td>OSMC</td>
<td>173.00</td>
<td>154.35</td>
</tr>
<tr>
<td>OFRSMC</td>
<td>113.52</td>
<td>55.70</td>
</tr>
</tbody>
</table>

Fig. 7. Experimental results of FCIS in controlling the flexible PET/ITO substrates at 2 cm bending radius. The red solid line is the system output based on SMC with FOE, the blue solid line is the system output based on OSMC, and the green solid line is the system output based on OFRSMC. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 8. Experimental results of FCIS in controlling the flexible substrate at 4 cm bending radius repeatedly based on (a) SMC with FOE, (b) OSMC and (c) OFRSMC.
OSMC, OFRSMC can overcome variable parameters, nonlinearity and measurement noise to achieve a better bending-radius control. Since experimental and simulation results are in good agreement, it is concluded that the OFRSMC outperforms the other two methods. Therefore, it is certain that the OFRSMC method is capable of bending-radius control on FCIS for fast and robustly measuring electrical characteristics in bending.

### 4.2. Measurement results of electrical characteristics of flexible substrate in bending

According to the experimental results given in Section 4.1, FCIS could be utilized for measuring bending characteristics of flexible electronics under different bending radii, as it successfully provides a more stable and robust bending condition by using the OFRSMC method for measuring electrical characteristics of flexible electronics.

Therefore, in order to measure electrical characteristics of flexible substrate under different radii of curvature, a commercially available ITO-coated PET (OCTM 100) from CPFilms Inc. is utilized. It has 50 nm think ITO on 125 μm think PET sheet. A laser writer is used in this work to make different widths of the line pattern on the PET/ITO sample. For example, Fig. 9 shows a 5 dpi width of the line pattern on a PET/ITO sample. The length and width of the line pattern are 152 mm and 5 mm, respectively. Therefore, by using FCIS based on the OFRSMC the method of 4-point-probe electrical-resistance measurement is applied for measuring the electrical resistance of the line pattern in flat and 2 cm bending radius conditions.

There are two types for bending a flexible substrate. One is the compressive bending, which means the bending direction makes ITO layers compressed, whereas the other is tensile bending. For example, Fig. 10(a), (b), and (c) depicts that ITO layers in flexible PET/ITO sample are in flat, compressive bending, and tensile bending, respectively. A one-time bending cycle by using FCIS based on the OFRSMC is to control from flat, 2 cm bending radius, and return to flat. Therefore, a 50 dpi PET/ITO sample was measured in flat and 2 cm compressive-bending radius conditions, respectively, for bending 11,000 times by using FCIS based on the OFRSMC, as shown in Table 3. In addition, this study...
measures the electrical resistance for different widths of the line pattern on the sample up to 11,000 bending times. Fig. 11 depicts the change-difference and change-rate of electrical resistance after 11,000 times tensile and compressive bending for 5, 10, 20, 30, 40, and 50 dpi samples, respectively, where change-differences are calculated from the difference of the electrical resistance of the line pattern between before and after bending, and change-rates are calculated from the difference of the electrical resistance of the line pattern between before and after bending divided by the electrical resistance of the line pattern.

Table 3

<table>
<thead>
<tr>
<th>Bending times</th>
<th>Bending conditions</th>
<th>2 cm Bending radius</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flat</td>
<td>Change-rate of electrical resistance (%)</td>
</tr>
<tr>
<td>Bending radius (cm)</td>
<td>Resistance (kΩ)</td>
<td>Change-rate of electrical resistance (%)</td>
</tr>
<tr>
<td>0</td>
<td>20.976</td>
<td>0.000</td>
</tr>
<tr>
<td>500</td>
<td>21.046</td>
<td>0.334</td>
</tr>
<tr>
<td>1000</td>
<td>21.063</td>
<td>0.415</td>
</tr>
<tr>
<td>2000</td>
<td>21.082</td>
<td>0.505</td>
</tr>
<tr>
<td>3000</td>
<td>21.100</td>
<td>0.591</td>
</tr>
<tr>
<td>4000</td>
<td>21.110</td>
<td>0.639</td>
</tr>
<tr>
<td>5000</td>
<td>21.117</td>
<td>0.672</td>
</tr>
<tr>
<td>6000</td>
<td>21.126</td>
<td>0.715</td>
</tr>
<tr>
<td>7000</td>
<td>21.129</td>
<td>0.729</td>
</tr>
<tr>
<td>8000</td>
<td>21.131</td>
<td>0.739</td>
</tr>
<tr>
<td>9000</td>
<td>21.135</td>
<td>0.758</td>
</tr>
<tr>
<td>10,000</td>
<td>21.140</td>
<td>0.782</td>
</tr>
<tr>
<td>11,000</td>
<td>21.145</td>
<td>0.806</td>
</tr>
</tbody>
</table>

Fig. 11. (a) Change-difference of electrical resistance and (b) change-rate of electrical resistance after 11,000 times tensile and compressive bending for 5, 10, 20, 30, 40, and 50 dpi PET/ITO samples, respectively.
before bending. According to measurement results, the following conclusions are drawn:

1. Table 1 depicts that the sample resistance with 2 cm bending radius is larger than the flat one, because the ITO-line pattern becomes thinner in bending. In addition, it also depicts that as the bending times increase, change-rates of electrical resistance in both flat and 2 cm bending radius conditions also increase.

2. In Table 1, both results of the flat and 2 cm bending radius conditions are similar.

3. Decreasing the width of the line pattern also increases change-rates of the resistance in both flat and 2 cm bending radius conditions.

4. As a consequence, tensile and compressive bending curve-fitting equations are respectively written as:

\[
C_d = 1.1148e^{0.117w} \quad (46)
\]

\[
C_d = 1.8368e^{-0.097w} \quad (47)
\]

where \(C_d\) is the change-difference of resistance between before and after 11,000 bending times, and \(w\) is the width of line patterns on the PET/ITO samples. According to Fig. 11 and both Eqs. (46) and (47), the relationship between the width of the line pattern and the change-rate of electrical resistance is exponential.

In addition, this study obtains change-rate curve-fitting equations for tensile and compressive bendings:

\[
C_f = 0.0613e^{0.088w} \quad (48)
\]

\[
C_f = 0.0806e^{-0.0482w} \quad (49)
\]

where \(C_e\) is the change-rate of electrical resistance between before and after 11,000 bending times, and \(w\) is the width of line patterns on the PET/ITO samples. From Fig. 11 and both Eqs. (48) and (49), the relationship between the width of the line patterns and the change-rate of electrical resistance is also exponential.

5. According to measurement results for two bending types, the increasing rate for the change-difference or the change-rate of electrical resistance in tensile bending is higher than in compressive bending when decreasing the line pattern width.

5. Conclusions

This study has presented a stable, robust, and fast-response bending-radius condition of flexible substrates by using FCIS based on the OFRSMC. This study also has measured electrical properties of flexible PET/ITO substrates up to 11,000 bending times by using FCIS based on the OFRSMC. Measurement results lead to the exponential relation between the width of the line pattern and the change-difference or the change-rate of electrical resistance in tensile or compressive bending. The relationship will be an important reference for flexible electronics research in the future. Accordingly, FCIS is a good tool for inspections of flexible displays under bending. In addition to the electrical property, the present system can also be utilized to measure other properties in bending, e.g., mechanical and optical characteristics, depicted on flexible electronics.

References


